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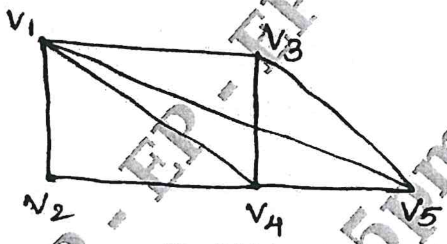
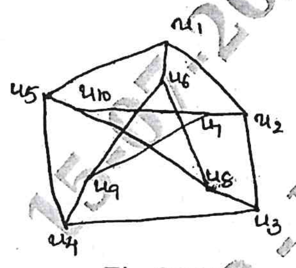
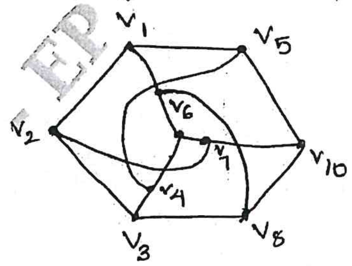
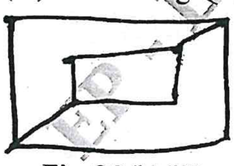
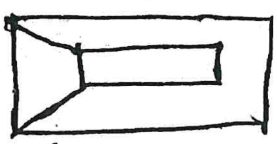
**BCS405B**

**Fourth Semester B.E/B.Tech. Degree Examination, June/July 2025**

# Graph Theory

Time: 3 hrs. Max. Marks:100

**Note:** *1. Answer any FIVE full questions, choosing ONE full question from each module.*
*2. M : Marks , L: Bloom's level , C: Course outcomes.*

| | | **Module – 1** | **M** | **L** | **C** |
|---|---|---|---|---|---|
| 1 | a. | Consider the following graph G Fig.Q1(a). Write :<br>i) Open walk which is not a trail<br>ii) Trial which is not a path<br>iii) Closed walk which is a cycle<br>iv) Closed walk which is a circuit but not a cycle<br>v) Closed walk neither circuit nor cycle<br>vi) Path of length 4.<br><br><br>Fig.Q1(a) | 6 | L3 | CO1 |
| | b. | Define bipartite graph and complete bipartite graph can a bipartite graph have odd length cycles. Explain. | 7 | L1 | CO1 |
| | c. | Is there a simple graph with 1, 1, 3, 3, 3, 4, 6, 7 as the degree of vertices? Explain. | 7 | L3 | CO1 |
| | | **OR** | | | |
| 2 | a. | Define spanning subgraph and induced subgraph. Draw a complete graph G with 5 vertices and spanning subgraph and induced subgraph of G. | 6 | L1 | CO1 |
| | b. | Verify the following :<br>i) Fig.Q2(b)(i) and Fig.Q2(b)(ii) are isomorphic.<br><br><br>Fig.Q2(b)(i)<br><br><br>Fig.Q2(b)(ii)<br>ii) Fig.Q2(b)(iii) and Fig.Q2(b)(iv) are not isomorphic.<br><br>Fig.Q2(b)(iii)<br><br>Fig.Q2(b)(iv) | 7 | L2 | CO2 |
| | c. | A simple graph with n vertices and k components can have at most (n – k) (n – k + 1)/2 edges. | 7 | L3 | CO2 |

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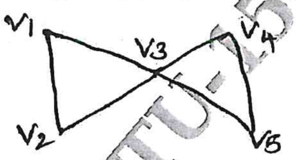
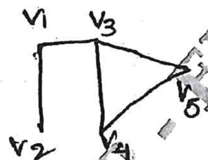
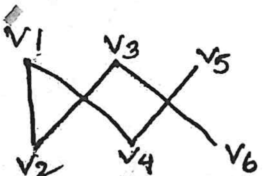
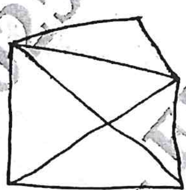
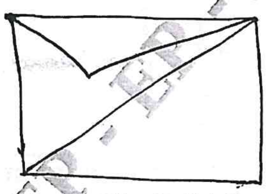
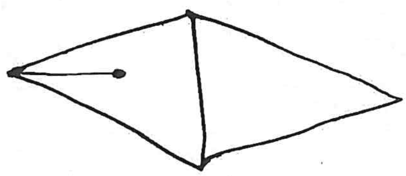
| | | **Module – 2** | | | |
| 3 | a. | By specifying the walk draw two Euler graphs and unicursal graph. | 6 | L2 | CO1 |
| | b. | If all the vertices in a connected graph G are of even degree, then show that G is Eulerian. | 7 | L3 | CO2 |
| | c. | Define and find union, intersection and ring sum of $K_{2,3}$ and $K_{3,3}$. | 7 | L1 | CO2 |
| | | **OR** | | | |
| 4 | a. | i) Define reflexive relation, symmetric relation and transitive relation<br>ii) Draw a symmetric graph and complete asymmetric graph. | 6 | L1<br>L2 | CO1<br>CO1 |
| | b. | Distinguish between Hamiltonian graph and Eulerian graph with two examples by specifying the walk. | 7 | L2 | CO2 |
| | c. | Prove that a connected graph G has an Euler circuit if and only if G can be decomposed into edge-disjoint cycles. | 7 | L3 | CO2 |
| | | **Module – 3** | | | |
| 5 | a. | Prove that a tree with n vertices has n-1 edges. | 6 | L3 | CO1 |
| | b. | i) Prove that a graph is connected if and only if it has a spanning tree<br>ii) Identify cut vertices if any in graph Fig.Q5(b)(i), Fig.Q5(b)(ii), Fig.Q5(b)(iii).<br><br>Fig.Q5(b)(i)<br><br>Fig.Q5(b)(ii)<br><br>Fig.Q5(b)(iii) | 7 | L3<br>L2 | CO2<br>CO2 |
| | c. | Show that for any graph G, the vertex connectivity cannot exceed the edge connectivity and edge connectivity cannot exceed the degree of the vertex with the smallest degree in G. | 7 | L3 | CO3 |
| | | **OR** | | | |
| 6 | a. | Prove that a connected graph G is a tree if and only if there is one and only one path between every pair of vertices. | 6 | L3 | CO2 |
| | b. | Define a tree and forest. Prove that with two or more vertices in a tree, there are at last two pendent vertices. | 7 | L1 | CO1 |
| | c. | Show that a Hamiltonian path is a spanning tree. Draw all the spanning trees of the graph Fig.Q6(c).<br><br>Fig.Q6(c) | 7 | L2 | CO2 |
| | | **Module – 4** | | | |
| 7 | a. | i) State Kuratowski's theorem and draw Kuratowski's two graphs<br>ii) Draw planar graphs of : i) Order 5 and size 8   ii) Order 6 and size 12. | 6 | L1<br>L3 | CO2<br>CO2 |
| | b. | Show that a connected planar graph with n vertices and e-edges has e-n+2 regions. | 7 | L3 | CO2 |
| | c. | Draw the geometric dual of graphs Fig.Q7(c)(i) and Fig.Q7(c)(ii).<br><br>Fig.Q7(c)(i)<br><br>Fig.Q7(c)(ii) | 7 | L2 | CO3 |

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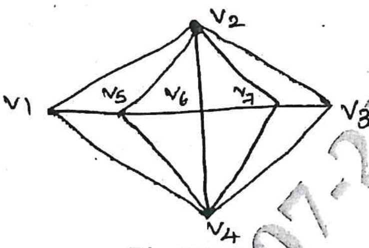
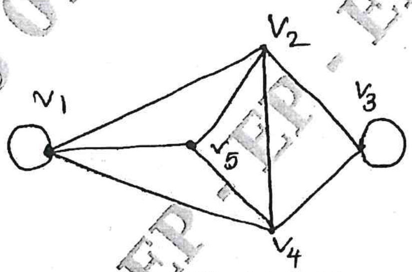
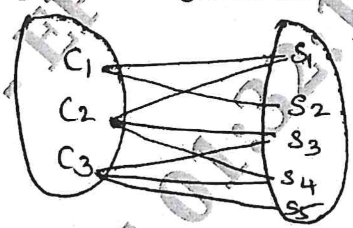
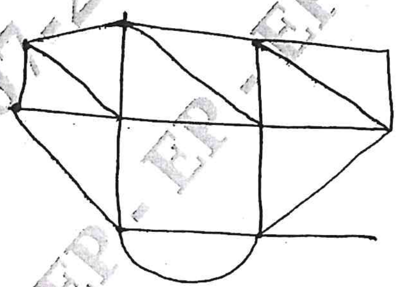
| | | **OR** | | | |
| **8** | a. | i) Show that Kuratowski's first graph $K_5$ isnon planar<br>ii) Show that every connected simple graph G contains a vertex of degree less than 6. | 6 | L2<br>L2 | CO2<br>CO2 |
| | b. | If G is a simple planar graph with at least three vertices then show that :<br>(i) $e \leq 3n - 6$ ii) $e \leq 2n - 4$ if G is triangle free. | 7 | L3 | CO2 |
| | c. | Write down adjacency matrix, path matrix and circuit matrix for the given graphs Fig.Q8(c)(i) and Fig.Q8(c)(ii).<br><br>Fig.Q8(c)(i)<br><br>Fig.Q8(c)(ii) | 7 | L2 | CO3 |
| | | **Module – 5** | | | |
| **9** | a. | Prove that a graph with at least one edge is 2-chromatic if and only if it has no circuits of odd length. | 6 | L3 | CO2 |
| | b. | Define chromatic number. Find chromatic polynomial of $C_4$ of length 4. | 7 | L2 | CO3 |
| | c. | State and prove 5 colour problems. | 7 | L3 | CO2 |
| | | **OR** | | | |
| **10** | a. | Prove that every connected simple planner graph is 6-colourable. | 6 | L3 | CO3 |
| | b. | Define matching and complete matching. Find the two complete matching of :<br><br>Fig.Q10(b) | 7 | L1 | CO2 |
| | c. | Define covering and minimal covering of a graph. Obtain two minimal covering from the given graph.<br><br>Fig.Q10(c) | 7 | L2 | CO3 |

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